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## TRADE-OFF BETWEEN MULTIPLE CRITERIA IN SMART HOME CONTROL SYSTEM DESIGN

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**Abstract.** *The successful automation of a smart home relies on the ability of the smart home control system to organize, process, and analyze different sources of information, according to several criteria. Because of variety of key design criteria that every smart home of the future should meet, the main challenge is the trade-off between them in uncertain environment. In this paper, a problem of smart home design has been solved using the methodology based on multiplicative form of multi-attribute utility theory. Aggregated functions describing different smart home alternatives are compared using stochastic dominance principle. The aggregation of different criteria has been performed through their numerical convolution, unlike usual approach of pairwise comparison, allowing only the additive form of aggregation of individual criteria. The methodology is illustrated on the smart home controller parameter setting.*

**Key words:** MAUT, decision making, multi criteria analysis, smart home, stochastic dominance

### 1. INTRODUCTION

Making a home smart means that residents move around safely and easily, economizing and using resources more efficiently. In order to accomplish these multiple tasks, a smart home must be equipped with technology that observes the residents and provides proactive services. With the increase of inexpensive sensors, communication equipment and embedded processors, smart homes are equipped with a large amount of sensors that use the acquired data on the activities and behaviors of its residents and consequently - perform appropriate control actions [1]. The successful automation of a smart home relies on the ability of the smart home control system to organize, process, and analyze different sources of information according to different

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criteria defined by the user. To this end, a strong and formal support to the multi-criteria decision is central to the smart home controller design and setting.

As far as smart home functionality is concerned, there are at least four major key design requirements that every smart home of the future should meet [2]:

- User-friendliness: a functionality must be comfortable and helpful to (often non-technical) home occupants.
- Intelligence for the most basic and sensible functions (such as turning on lights when coming, and turning them off when leaving home), requiring complex information processing of diverse information sources.
- Non-intrusiveness: the ability of the system to operate in the background, not bothering occupants by the proliferation of queries.
- Security and its accompanying factor, privacy, are extremely important for the adoption of any smart home system.

The trade-off between these criteria is necessary on all hierarchical levels of smart home design, selection and operation. We do not know what mix of sensors is optimal for a particular group or individual, and how to appropriately control, summarize and present information collected to different stakeholders. A series of technical and social challenges need to be addressed before sensor technologies can be successfully integrated according to the occupant's attitude to different criteria. Besides the presence of multiple criteria, another challenge in front of intelligent building and smart home automation is the great uncertainty due to the stochastic nature of renewable energy sources.

In this paper, the methodology for discrete stochastic multiple criteria decision making problem in smart home system design, with different types of tradeoffs among criteria has been applied for the smart home design selection problem. The advantage of this approach is the usage of compensatory aggregation, which is more suitable for conflicting criteria or the human aggregation behavior. The proposed methodology is based on numerical convolution of criteria probability distribution functions, according to different types of criteria aggregation. Alternatives are ranked according to the stochastic dominance (SD) rules.

The contribution of this paper is the introduction of new decision support tool which is more adapted to the smart home design faced with uncertainties and necessary trade-off between different criteria and different stakeholders. The methodology can be used for various problems in the smart home design, including the sensor disposition, parameter setting, functionality selection etc. Unlike previous multi-criteria approach, compensatory aggregation adapted to the human behavior has been applied.

The paper is organized in the following way. After the literature review of the current state of the problem, the methodology for stochastic multi criteria decision making (SMCDM) is presented, describing each step of the methodology: definition of the type of the criteria aggregation, numerical convolution of aggregated utility probability distributions and the application of SD rules for the ranking of alternatives. The methodology is illustrated on the choice of the smart home control parameter settings and finally, conclusions and further research directions are presented.

## 2. LITERATURE REVIEW

Generally, a home that is designed according to smart and sustainable home principle has to meet occupant's needs through all stages of their life. Previous work on smart home system design has been generally focused on a specific problem area such as information correlation or hardware [3], [4]. In [5], authors review sensor technology used in smart homes focusing on environment and infrastructure mediated sensing. In [6]-[9] smart home technology is a support for people with reduced capabilities due to aging or disability. Requirements generated from considerations of social, environmental, and economic issues for high efficient energy-saving building systems in compliance with building codes and regulations were analyzed in [10], [11]. Focusing on specific design problem, authors did not take on a holistic system and multi-criteria engineering view.

In [12], the general controller system design procedure based on evolutionary multiobjective optimisation (EMO) is presented, with the comprehensive review of other multi-objective design procedures. An extensive list of requirements for composition of smart home application has been provided in [13] and [14], where requirements are clustered in seven categories, each of which consisting of three to five requirements, including:

- Simplicity: describing the complexity of application development, involving the interaction between the system and the application developer.
- Modeling: requirements that affect the way the smart home applications can be modeled.
- Time: the ability to impose timing constraints
- Mobility: including both mobile devices and changes in the system
- Technical requirement for a composition solution
- Security, Safety and Privacy
- Miscellaneous, containing all requirements that do not match the other categories.

With the diversification of criteria and the increased number of stakeholders engaged in smart home realization, the need for multiobjective and multicriteria approach emerged. Starting from the redesign of building automation systems [15], various applications of multiobjective optimization of control systems were introduced, like the controller adjustment and controller parameter selection [16]. In [17] fuzzy AHP multicriteria analysis of key performance indicators related to the smart grid efficiency, as the key factor of any energy management system implementation have been analyzed. However in all of mentioned approaches the multiobjective problem is normalized and converted to a single-objective optimization with deterministic state of nature concerning the consequences of different alternatives.

Although the authors present a multi-criteria decision-making model using the analytic network process to evaluate the lifespan energy efficiency of intelligent buildings, the trade-off between different criteria has not been taken into account in all mentioned approach.

As stated before, stochastic nature of renewable sources integrated in intelligent buildings requires stochastic predictors [15], [18]. However, authors conclude that the current technology is still not mature enough for cost-effective usage in most of the real-world scenarios.

One of the prominent stochastic and multicriteria methodology - *SMCDM* is used for selecting alternatives associated with multiple criteria, where consequences of alternatives with respect to criteria are in the form of random variables. There are three general methods to solve *SMCDM* problem: 1) outranking methods using confidence indices on alternative

pairwise comparisons with respect to each criterion [19], 2) Data Envelopment Analysis [20] and 3) stochastic multi-objective acceptability analysis (SMAA) [21]. Methods using stochastic processes and *SD* rules generally include two processes [22], [23]: comparison and selection. The comparison serves to identify whether there exists a *SD* relation for comparison of any pair of alternatives using *SD* rules, while the selection is to rank alternatives based on the determined *SD* relations using Rough Set Theory or interactive procedures [24], [25]. In stochastic multi attribute analysis (SMAA) or group decision-making analysis, both criterion values and criterion weights are uncertain but the usage of more complex utility functions together with the correlation between attributes remained neglected.

So far, *SMCDM* problems were exclusively related to the additive form of utility functions, with evaluations  $e_{ij}$  taken as utility values. In [26] a range of simulated problem settings is used to show that using an additive aggregation when preferences actually follow a multiplicative model may often only have minor impacts on results. However, for many decision problems, including the various smart home design phases, estimated parameters are inconsistent with the linear additive case and are strongly favoring the multiplicative functional form. Furthermore, decision makers tend to partially compensate between criteria, instead of trying to satisfy them simultaneously, emphasizing the need for the multiplicative functional form. In [27], a new methodology for the multidimensional risk assessment, based on stochastic multiattribute theory has been presented. This methodology encompasses simultaneously: the multi criteria decision problem, stochastic nature of criteria outcomes and trade-off between them depending on decision maker preferences, making it the candidate for the smart home controller design problems.

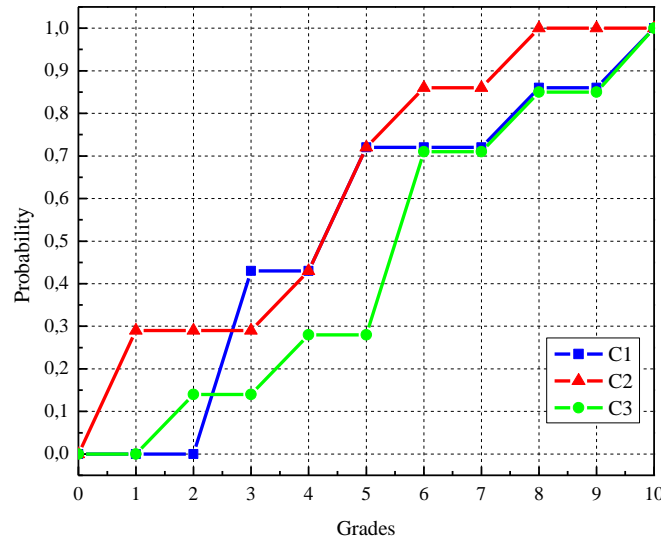
### 3. METHODOLOGY

The main challenge in the smart home control system design is the presence of great number of different stakeholders, with different and often opposite preferences. For the sake of illustration, suppose that seven persons evaluate different alternatives for indoor temperature setting (e.g. 20° C) over the set of three criteria: comfort ( $C_1$ ), ecology ( $C_2$ ) and energy costs ( $C_3$ ), on a scale of ten (1 - the worst, 10 - the best). The evaluations of  $i$ -th alternative are expressed in the form of the discrete probability distribution as shown in Table 1.

**Table 1** Evaluation distribution of three criteria for an indoor temperature setting value

Scores	Criteria		
	C1	C2	C3
1	0	2/7	0
2	0	0	1/7
3	3/7	0	0
4	0	1/7	1/7
5	2/7	2/7	0
6	0	1/7	3/7
7	0	0	0
8	1/7	1/7	1/7
9	0	0	0
10	1/7	0	1/7

The graphical representation of appropriate cumulative distribution functions is given on Figure 1.



**Fig. 1** The cumulative distribution functions of three criteria evaluations

The problem is how to make a trade-off between these criteria and how to choose the required temperature to satisfy all occupants' preferences. Furthermore, on other levels of smart home design or operation, the same problem of multi-criteria decision analysis in presence of group of decision makers, or uncertain environment still exists. The methodology proposed in this paper for solving this problem is based on multi-attribute utility theory (MAUT) and numerical convolution of probability distribution. The reader is referred to the article [27] for the detailed explanation of the methodology, but the key points will be explained in the sequel.

A decision problem is consisting of  $n$  alternatives denoted by  $a_i$ ,  $i \in \{1, \dots, n\}$  each evaluated on  $m$  criteria denoted by  $c_j$ ,  $j \in \{1, \dots, m\}$ . Let  $e_{ij}$  be the evaluation of  $a_i$  in terms of criterion  $c_j$ , according to some suitable performance measure. We focus on decision making situations in which the values of  $e_{ij}$  for each  $i$  are not known with certainty for all  $j$ , but follow some distribution function  $f(e_{ij})$ . This formulation is known as Alternatives, Attributes (Criteria), Evaluators (AAE or ACE) model.

The process of selecting the optimal smart home design is performed in following steps:

- Identification of different alternatives and criteria.
- Formation of individual criteria probability distribution functions.
- The aggregated probability distribution formation by the numerical convolution of marginal probability distributions.
- SD evaluation on aggregated probability functions

### 3.1. Criteria aggregation

The following three types of aggregation of criteria are used most commonly in decision making: conjunctive, disjunctive and compensatory. Conjunctive aggregation implies simultaneous satisfaction of all decision criteria, while the disjunctive aggregation implies full

compensation amongst them. The compensatory aggregation is more suitable for human aggregation behavior. Among the great number of different compensatory aggregation operators, multiplicative multi-attribute utility function proved to be the most suitable for practical engineering applications. It is shown that if the additive independence condition is verified, a multi-attribute comparison of two actions can be decomposed to one-attribute comparisons. If mutual utility independence exists, the multi-attribute utility function is of the following form [28]:

$$U(x_1, x_2, \dots, x_n) = \frac{\prod_i (1 + K k_i u_i(x_i)) - 1}{K} \quad (1)$$

Here,

$u_i(x_i)$  – the single-attribute utility value for attribute  $i$  with value  $x_i$  (ranges from 0 to 1),

$k_i = a$  – parameter from the trade-off for component  $i$ , for all  $i$ , and

$K = a$  – normalization constant, ensuring that the utility values are scaled over the component range space between 0 and 1.

One method to determine the multiplicative function (1) is to measure each  $u(x)$ , determine the  $k_j$  values, and find the  $K$  value by iteratively solving (2).

$$1 + K = \prod_{i=1}^n (1 + K \cdot k_i) \quad (2)$$

Parameter  $K$  is related to parameters  $k_i$  as follows:

$$\text{if } \sum_{i=1}^n k_i > 1, \text{ then } 1 < K < 0, \quad (3)$$

$$\text{if } \sum_{i=1}^n k_i = 1, \text{ then } K = 0, \text{ and the additive model holds,} \quad (4)$$

$$\text{if } \sum_{i=1}^n k_i < 1, \text{ then } K > 0. \quad (5)$$

The overall utility function actually reflects three different types of interactions between individual criteria. In the compensatory case, performance of one criterion makes up for the lack of performance by other criteria, while in the additive case, it does not interact with the value of the other criteria. In the complementary case, a good performance by one criterion is less important than balanced performance across the criteria.

### 3.2. SMCDM with compensatory aggregation

The main idea of the proposed methodology is to compare different alternatives using a pragmatic aggregation function for combining the single-utility functions from each of the system components. This comparison is possible because of equivalence of rules for multivariate utility function  $u = u(x_1, x_2, \dots, x_n)$  and univariate utility function defined on multivariate outcome space  $u = u^s(P(x_1, x_2, \dots, x_n))$ .

In order to make the ranking of alternatives more practical, the convolution of these probability distributions to enable the comparison of only one distribution function per alternative is proposed. After the new, aggregated probability distribution has been built for every alternative, the ranking of alternative is performed by SD rules explained in the Appendix. Different uncertainty types, like outcomes and weighting factors can be simultaneously handled by the convolution principle.

The four step methodology of alternative ranking is based on the multiplicative utility function as a combination of suggested criteria and decision maker attitude towards risk, numerical convolution of individual distribution functions and SD principle.

### 3.3. Aggregation of utility distribution functions

Let  $X$  and  $Y$  be two independent integer-valued random variables, with distribution functions  $f_X$  and  $f_Y$  respectively. Then the convolution of  $f_X$  and  $f_Y$  is the distribution function  $f_Z$  given by:

$$f_Z(j) = \sum_k f_X(k) \cdot f_Y(j-k), \quad (6)$$

for  $j = -\infty, \dots, +\infty$ . The function  $f_Z(j)$  is the distribution function of the random variable  $Z = X + Y$ .

In [29], an efficient algorithm for computing the distributions of sums of discrete random variables is presented. However, multiplicative form of utility function requires other convolution type. In the proposed methodology, the computational procedure is extended to different forms of aggregating function and speeded up by the reduction of dimensions of arrays  $P$  and  $Z$  to the number of evaluation grades, according to the following algorithm. For  $n$  criteria, and  $m$  number of evaluation grades, dimension of output array is reduced to  $m$  instead of  $m \times n$ . The algorithm for the discrete convolution algorithm is given below:

*Input:*  $F(x_1, \dots, x_n)$  – multi-attribute utility function;  $m$  – number of evaluation grades;  $p(x_i = j)$  – probability that variable  $i$  takes the value  $j$ ,  $j = (1, m)$ .

▪ For  $i = 1$  to  $m$   
For  $j = 1$  to  $m$

...

For  $n = 1$  to  $m$

Calculate  $F(x_1 = i, x_2 = j, \dots, x_n = n)$

$z = \text{integer}(F)$

[discretization of  $F$ ]

$p(z) = p(z) + [p(x_1) \cdot p(x_2) \cdot \dots \cdot p(x_n)]$

*Output:*  $Z$

[dimension  $m$ ]

The cumulative distribution function of aggregated random variable  $U$  is given by (7).

$$F_X(x) = P(X \leq x) = \sum_{u \leq x} P(X = u) = \sum_{u \leq x} f_X(u), \quad (7)$$

The comparison of different CDFs corresponding to aggregated utility function is now possible with the SD principle. The first step is the formation of aggregation function based on suggested criteria and DM attitude towards risk. In the second step, using the numerical convolution of individual criterion probability distribution functions, an aggregated probability distribution is derived. In the third step, using SD rules and SD degree values, a dominance matrix is formed.

The final step in this methodology is the alternative ranking based on the results of the dominance matrix. Two types of dominance matrices will be used in this methodology: the first one obtained by the three types of stochastic dominance. Using the first, second or third degree stochastic dominance rule, the appropriate type of the dominance matrix is obtained, where the elements of the dominance matrix are defined in the following way:

$sd_{ij} = 1$ , if  $F_{A_i} SD_h F_{A_j}$ , otherwise,  $sd_{ij} = 0$ ,  $h \in \{1, 2, 3\}$ .

The methodology will be illustrated on the example of smart home controller parameter selection concerning four criteria explained in the introductory section.

#### 4. CASE STUDY

We consider one of many possible smart home functions: the blackout prevention for the smart house, where the smart meter measures the real-time power levels of appliances and send this information to smart home control system. The control system calculates the remaining available power, and send this information to the appliances, but with a time delay.

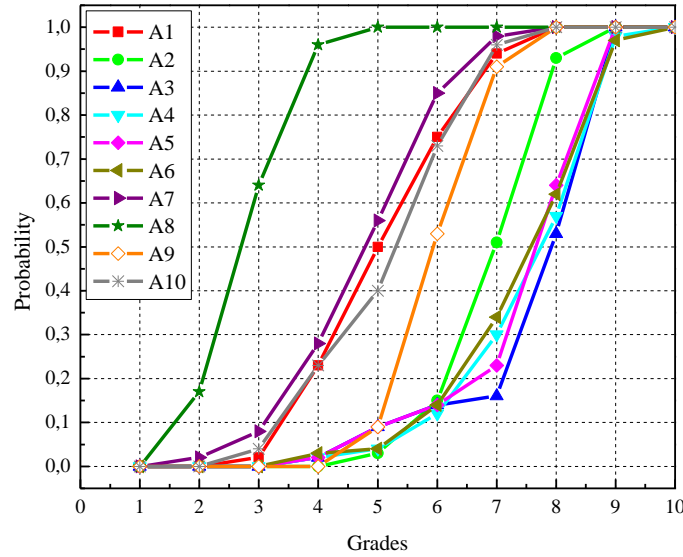
**Table 2.** Expert's evaluation of alternatives

Criteria	Scores	Alternatives									
		A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>	A <sub>9</sub>	A <sub>10</sub>
C <sub>1</sub>	1	0	0	0	1/7	0	1/7	1/7	1/7	0	0
	2	3/7	1/7	0	0	0	0	0	2/7	0	1/7
	3	1/7	0	0	0	1/7	0	0	2/7	0	2/7
	4	0	2/7	0	0	0	0	0	1/7	0	2/7
	5	2/7	1/7	3/7	1/7	0	0	3/7	1/7	2/7	1/7
	6	0	2/7	1/7	0	2/7	0	1/7	0	1/7	0
	7	1/7	0	1/7	0	2/7	1/7	0	0	3/7	1/7
	8	0	1/7	2/7	1/7	0	4/7	1/7	0	1/7	0
	9	0	0	0	4/7	2/7	0	0	0	0	0
	10	0	0	0	0	0	2/7	1/7	0	0	0
C <sub>2</sub>	1	0	1/7	1/7	0	0	0	1/7	3/7	0	0
	2	2/7	0	0	0	0	0	3/7	3/7	0	1/7
	3	1/7	0	0	1/7	0	4/7	1/7	0	1/7	0
	4	0	0	0	1/7	0	0	0	1/7	1/7	0
	5	2/7	0	0	0	1/7	0	1/7	0	0	0
	6	0	1/7	1/7	1/7	2/7	0	1/7	0	1/7	0
	7	0	1/7	0	0	1/7	1/7	0	0	4/7	2/7
	8	1/7	1/7	2/7	3/7	2/7	2/7	0	0	0	3/7
	9	1/7	3/7	1/7	1/7	1/7	0	0	0	0	0
	10	0	0	2/7	0	0	0	0	0	0	1/7
C <sub>3</sub>	1	0	0	1/7	0	1/7	0	0	2/7	0	1/7
	2	0	0	0	0	0	0	3/7	1/7	0	2/7
	3	1/7	0	0	1/7	0	0	1/7	4/7	1/7	0
	4	3/7	0	0	0	0	1/7	1/7	0	2/7	0
	5	0	1/7	0	0	0	1/7	2/7	0	2/7	0
	6	1/7	0	0	0	0	0	0	0	0	2/7
	7	0	1/7	0	1/7	0	0	0	0	2/7	2/7
	8	1/7	2/7	0	2/7	3/7	2/7	0	0	0	0
	9	1/7	3/7	2/7	1/7	1/7	1/7	0	0	0	0
	10	0	0	4/7	2/7	2/7	2/7	0	0	0	0
C <sub>4</sub>	1	0	1/7	0	1/7	0	0	0	2/7	0	0
	2	0	0	0	0	0	0	0	0	1/7	0
	3	3/7	0	0	0	0	0	1/7	0	0	0
	4	0	0	0	0	0	0	0	1/7	1/7	0
	5	2/7	0	0	0	0	1/7	1/7	2/7	0	0
	6	0	0	0	0	1/7	1/7	0	1/7	3/7	3/7
	7	0	0	1/7	0	1/7	1/7	0	0	0	1/7
	8	1/7	2/7	4/7	0	3/7	2/7	3/7	1/7	1/7	1/7
	9	0	2/7	0	1/7	1/7	1/7	1/7	0	0	1/7
	10	1/7	2/7	2/7	5/7	1/7	1/7	1/7	0	1/7	1/7



Let suppose that we can build 10 alternatives with different combination of appliances and times for their disconnection, directly affecting all of four criteria concerning the smart home functionality requirements. In the problem, the set of ten alternatives is  $(A_1, A_2, \dots, A_{10})$  and the criteria considered include: user friendliness  $C_1$ , intelligence complexity  $C_2$ , non-intrusiveness  $C_3$  and security  $C_4$ . Suppose that seven persons provide evaluations on the alternatives with respect to the criteria on a scale of ten (1 - the worst, 10 - the best). The complete table of probability distributions of expert's evaluation is presented in Table 2. The similar problem, which served as as basis for our analysis is given in [23],[25],[31].

The proposed method is illustrated with the multiplicative utility function of four existing criteria. Using the expression (1), the aggregated utility function is obtained with the supposed weighting factors:  $k_1 = 0.5, k_2 = 0.2, k_3 = 0.57, k_4 = 0.09, K = -0.686$ . Applying the numerical convolution of four criteria probability functions, ten aggregated probability distributions are obtained, represented on Figure 2.



**Fig. 2.** Aggregated probability distributions for ten different alternatives

Using the stochastic dominance degree, the dominance matrix is obtained (8). As explained in the Appendix the premise of calculating the *SDD* on a pair of alternatives is that there must be the *SD* relation on the pair of alternatives. The matrix element *SDD* (*i,j*) represents the degree of the dominance of the alternative *i* over the alternative *j*.

$$SDD = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.30 & 0 & 0 \\ 0.33 & 0 & 0 & 0 & 0 & 0 & 0.37 & 0.53 & 0.24 & 0.32 \\ 0.46 & 0.19 & 0 & 0.03 & 0.05 & 0.06 & 0.49 & 0.62 & 0.35 & 0.45 \\ 0.44 & 0.16 & 0 & 0 & 0.02 & 0.03 & 0.47 & 0.61 & 0.34 & 0.43 \\ 0.43 & 0.14 & 0 & 0 & 0 & 0.01 & 0.46 & 0.60 & 0.31 & 0.42 \\ 0.42 & 0.13 & 0 & 0 & 0 & 0 & 0.45 & 0.60 & 0.24 & 0.41 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.26 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.17 & 0 & 0 & 0 & 0 & 0 & 0.22 & 0.42 & 0 & 0.16 \\ 0.01 & 0 & 0 & 0 & 0 & 0 & 0.07 & 0.31 & 0 & 0 \end{bmatrix}, \quad (8)$$

As the final step, the ranking of alternatives is performed based on the values from the dominance matrix.

$$A_3 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_9 \succ A_{10} \succ A_1 \succ A_7 \succ A_8, \quad (9)$$

The power and flexibility of the proposed method is illustrated on the same example, with additive utility function of four existing criteria and the criterion weight vector  $w = [0.09; 0.55; 0.27; 0.09]$ , as proposed in the original example in [23]. The comparison of alternative ranking obtained from the previous matrix with three already mentioned methods is given in Table 3.

**Table 3.** Different alternative ranking methods comparison

Method	Ranking
Proposed method	$A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_6 \succ A_{10} \succ A_9 \succ A_1 \succ A_7 \succ A_8$
Zhang et al.	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_6 \succ A_{10} \succ A_9 \succ A_1 \succ A_7 \succ A_8$
Zaras and Martel's	$A_3, A_4 \succ A_2, A_5 \succ A_6, A_{10}, A_9 \succ A_1, A_7 \succ A_8$
Nowak	$A_3 \succ A_2 \succ A_4, A_5 \succ A_6 \succ A_9, A_{10} \succ A_1 \succ A_7 \succ A_8$

The proposed method gives the same results as the method of Zhang et al. [31]. However, instead of pairwise comparison of alternatives for individual criterion the result is obtained in only three steps explained above. The simulation is performed on Intel(R)Xeon(R) CPU E5-26670 @ 2.90 GHz processor with 32 GB RAM. The total time for the simulation was 1.3 sec that proves the suitability of the method in real time smart home applications.

## 5. CONCLUDING REMARKS

Proper smart home design depends on human judgment in great extent. In many practical applications, criteria in different stages of smart home design can be presented as random variables with appropriate discrete probability density function. These applications include, but are not limited to the scheduling of appliances in the presence of stochastic renewable production, control parameter selection and the choice of control strategy in uncertain

environment. In this paper, a problem of optimal design alternative selection has been solved with enhanced SMCDM methodology, based on numerical convolution of criteria probability distribution functions, according to multiplicative aggregation form. The methodology is based on multiplicative form of multi-attribute utility theory, which proved to be suitable for the modeling of human behavior in front of opposite criteria. The ranking of alternative is performed by the stochastic dominance degree.

Because of variety of key design criteria that every smart home should meet, and the trade-off between them in uncertain environment, this method proved to be efficient, unlike usual approach of pairwise comparison, allowing only the additive form of aggregation of individual criteria. In previous methodologies, the decision maker risk attitude is taken into account only at individual level of criterion comparison, while this attitude can be directly incorporated in the model with the different compensatory aggregators.

Together with the multiple uncertainties of evaluations and weighting factors, the problem of group decision making in smart home applications will be the focus of further researches of the possible application of this methodology.

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## APPENDIX

### STOCHASTIC DOMINANCE

In order to determine whether a relation of stochastic dominance holds between two distributions, the distributions are characterized by their cumulative distribution functions, or CDFs. Suppose that we consider two distributions  $A$  and  $B$ , characterized respectively by CDFs  $F_A$  and  $F_B$ . Then distribution  $B$  dominates distribution  $A$  stochastically at first order if, for any argument  $y$ ,  $F_A(y) \geq F_B(y)$ .

The *SD* rules can be fundamentally classified into two groups for two classes of utility functions. The first group is for increasing concave utility function and includes first degree stochastic dominance, second degree stochastic dominance and third degree stochastic dominance. These rules can be applied for modeling risk averse preferences.

**Definition 1.** Let  $a$  and  $b$  ( $a < b$ ) be two real numbers,  $X$  and  $Y$  be two random variables,  $F(x)$  and  $G(x)$  be cumulative distribution functions of  $X$  and  $Y$ , respectively. Let  $U_1$  include all the utility functions  $u$  for which  $u' \geq 0$ ,  $U_2$  include all the functions  $u$  for which  $u' \geq 0$  and  $u'' \leq 0$ ,  $U_3$  include all the functions  $u$  for which  $u' \geq 0$  and  $u'' \leq 0$  and  $u''' \geq 0$ .

Let  $E_F$  and  $E_G$  be the two expectations or the means, respectively. Let  $SD_1$ ,  $SD_2$  and  $SD_3$  denote first, second and third degree stochastic dominance, respectively. The *SD* rules are:

$F(x) SD_1 G(x)$  if and only if

$E_F(u(X)) \geq E_G(u(Y))$  for all  $u \in U_1$  with strict inequality for some  $u$ , or

$F(x) \leq G(x)$  for all  $x \in [a, b]$  with strict inequality for some  $x$ ;

$F(x) SD_2 G(x)$  if and only if

$E_F(u(X)) \geq E_G(u(Y))$  for all  $u \in U_2$  with strict inequality for some  $u$ , or

$\int_a^x F(t)dt \leq \int_a^x G(t)dt$  for all  $x \in [a, b]$  with strict inequality for some  $x$ ;

$F(x) SD_3 G(x)$  if and only if  $E_F(X) \geq E_G(Y)$

$E_F(u(X)) \geq E_G(u(Y))$  for all  $u \in U_3$  with strict inequality for some  $u$ , or

$\int_a^x \int_a^t F(z)dzdt \leq \int_a^x \int_a^t G(z)dzdt$  for all  $x \in [a, b]$  with strict inequality for some  $x$ ;

The second group of *SD* rules is for increasing convex utility function and includes first degree stochastic dominance, second inverse stochastic dominance, third inverse stochastic dominance of the first type and third inverse stochastic dominance of second type. These rules are equivalent to expected utility maximization rule for risk-seeking preferences.

**Definition 2.** In [30], a *SD* degree is defined, in the following way: if  $F(x) \cdot SD_h \cdot G(x)$ ,  $h \in \{1, 2, 3\}$  then the stochastic dominance degree *SDD* of  $F(x) \cdot SD \cdot G(x)$  is given by:

$$\psi(F(x)SD_h G(x)) = \frac{-\int [F(x) - G(x)]dx}{\int_{\Omega} G(x)dx}, h \in \{1, 2, 3\}, \Omega = \{x | x \in [a, b]\},$$

Both *SD* rules and *SD* degrees are used in the proposed methodology. According to [29], classes  $U_i$  ( $i = 1, 2, 3$ ) are identical to the following classes:

$$U = \left\{ \left\{ u(x, x, \dots, x) = \left\{ u(P(x, x, \dots, x)), u \in U \text{ and } P \in U \right\}, \text{ for each } i = 1, 2, 3, \right. \right.$$

$u^s$  is a single attribute utility function and  $P = P(x, x, \dots, x)$  a multivariate function, and  $U = U$  for  $i = 1, 2, 3$ .